Comment on a mechanism of
dynamical breaking of supersymmetry

Tomohiro Matsuda

Department of Physics,University of Tokyo

Bunkyo-ku,Tokyo 113,Japan

Abstract

We re-examine the so-called Nambu-Jona-Lasinio mechanism suggested by Song, Xu and Chin in breaking the supersymmetry in the Wess-Zumino model and show that this mechanism cannot be justified without assuming special effects between fermions. The fermion condensation suggested by them corresponds to an unstable vacuum configuration. As a result, there is no fermion condensation and no supersymmetry breaking in the model discussed by them.

1   Introduction

Recently it has been discussed in a series of papers by Song, Xu and Chin [1] that spontaneous supersymmetry breaking can be realized in a chiral symmetric model without adding a Fayet-Iliopoulos or O'Raifeartaigh term. In their analysis, the so-called Nambu-Jona-Lasinio mechanism was used, and they suggest that the fermion pair condensation induces a mass gap between supersymmetric partners. If their mechanism really works, it would open many possibilities in supersymmetric models. The purpose of the present paper is to present the short comings of their argument and clarify the physical backgrounds of it. The main point is very simple: They neglected the one-loop effects

1

of bosonic particles. Including these contributions correctly, we obtain the well-known one-loop effective potential, and their solution corresponds to an unstable configuration of this effective potential.

This paper is organized as follows. In section 2 we review the construction of an effective potential in the Wess-Zumino model. Then we re-examine the so-called NJL method proposed in[1] and clarify the physical backgrounds. Concluding remarks are also given in section 3.

2   Review of one-loop effective potential in WZ model

The analysis of supersymmetry breaking in the Wess-Zumino model is as old as the modern theory of supersymmetry[2]. Using a superfield method, Fujikawa and Lang[3] constructed a one-loop effective potential for the Wess-Zumino model and discussed the stability of the supersymmetric vacuum. Many authors, for example in[4], later discussed this and related topics.

For the notational convention, we use the two-component representation : By explicitly separating the vacuum expectation values of bosonic fields, we derive the one-loop effective potential by means of the tadpole method[5] instead of the direct evaluation of it[3].

The starting Wess-Zumino Lagrangian for a chiral super multiplet is given by

$$
\begin{aligned}
L &= \Phi^{+}\Phi|_{\theta\theta\overline{\theta}\overline{\theta}} + \left[\frac{1}{3!}\lambda\Phi^{3}|_{\theta\theta} + \frac{1}{2}m\Phi^{2}|_{\theta\theta} + h.c.\right] \\
&= i\partial_{m}\overline{\psi}\overline{\sigma}^{m}\psi + \overline{A}\Box A + \overline{F}F + \left[\frac{\lambda}{2}(A^{2}F - \psi\psi A) + m(AF - \frac{1}{2}\psi\psi) + h.c.\right] \quad (2.1)
\end{aligned}
$$

Shifting the bose fields of the theory in the fashion

$$
\begin{aligned}
A &\rightarrow A + a \\
F &\rightarrow F + f \quad (2.2)
\end{aligned}
$$

we obtain

$$
\begin{aligned}
L' &= i\partial_{m}\overline{\psi}\overline{\sigma}^{m}\psi + \overline{A}\Box A + \overline{F}F \\
&\quad + \left[\eta(AF - \frac{1}{2}\psi\psi) + \frac{\lambda}{2}(AAF - \psi\psi A) + \frac{\lambda}{2}fAA\right. \\
&\quad \left. + F(ma + \frac{\lambda}{2}a^{2} - \overline{f}) + A\eta f + h.c.\right] \quad (2.3)
\end{aligned}
$$

here we set

$$\eta = m + \lambda a. \tag{2.4}$$

Before calculating the effective potential, we should derive the propagators of the theory. Extracting the quadratic part of the boson fields,

$$
\begin{aligned}
S_0 &= \int d^4 x \frac{1}{2} \Phi^T A \Phi + \Phi^T J \\
&\quad \begin{cases} \Phi^T = (A, \overline{A}, F, \overline{F}) \\ J = (J, \overline{J}, K, \overline{K}) \end{cases} \\
&\quad A = \begin{pmatrix} -\lambda f & \Box & -\eta & 0 \\ \Box & -\lambda \overline{f} & 0 & \overline{\eta} \\ -\eta & 0 & 0 & 1 \\ 0 & -\overline{\eta} & 10 & \end{pmatrix}
\end{aligned}
\tag{2.5}
$$

the matrix A is easily inverted to obtain

$$
A^{-1} = \frac{1}{\Delta} \begin{pmatrix}
\lambda \overline{f} & \Box - \overline{\eta}\eta & \overline{\eta}(\Box - \overline{\eta}\eta) & \lambda \overline{f} f \\
\Box - \overline{\eta}\eta & \lambda f & \lambda f \overline{\eta} & \eta(\Box - \overline{\eta}\eta) \\
\overline{\eta}(\Box - \overline{\eta}\eta) & \lambda f \overline{\eta} & \lambda f \overline{\eta}\overline{\eta} & -\lambda^2 \overline{f} f + \Box(\Box - \overline{\eta}\eta) \\
\lambda \overline{f}\eta & \eta(\Box - \overline{\eta}\eta) & -\lambda^2 \overline{f} f + \Box(\Box - \overline{\eta}\eta) & \lambda \overline{f}\eta\eta
\end{pmatrix}
\tag{2.6}
$$

where

$$\Delta = (\Box - \overline{\eta}\eta)^2 - \lambda^2 \overline{f} f. \tag{2.7}$$

The tree level generating functional is now given by

$$ln Z_0 = -\frac{i}{2} \int d^4 x (J^T A^{-1} J). \tag{2.8}$$

Looking at $\frac{\delta^2 ln(Z_0)}{\delta J_1 \delta J_2}|_{J=0}$ the propagators of the theory are obtained directly.

Now let us derive the effective potential by means of the tadpole method. According to ref.[5], the following relation exists between the derivative of effective potential and 1PI tadpole.

$$\frac{dV(\phi_0)}{d\phi_0} = -\Gamma'^{(1)} \tag{2.9}$$

In this expression ϕ_0 means the vacuum expectation value(vev) of the field ϕ which can be any scalar field of the theory(in the present theory ϕ means A or F, and ϕ_0 means

3

a or f), $\Gamma'^{(1)}$ is the 1PI tadpole that is calculated after separating the vev and quantum fluctuation of the scalar fields as $\phi \to \phi + \phi_0$. So we use (2.3) to calculate $\Gamma'^{(1)}$. Using these relations, we obtain

$$\frac{V_0}{df} = ma + \frac{\lambda}{2}a^2 - \overline{f} \tag{2.10}$$

and

$$\frac{dV_1}{df} = -\frac{1}{2}\lambda^2 \int \frac{d^4p}{(2\pi)^4} \frac{\overline{f}}{(p^2 + \overline{\eta}\eta)^2 - \overline{f}f\lambda^2}. \tag{2.11}$$

After integration we get

$$V_0 = \left(ma + \frac{1}{2}\lambda a^2\right)f - \overline{f}f + P(\overline{f}, a, \overline{a}) \tag{2.12}$$

and

$$V_1 = \frac{1}{2}\int \frac{d^4p}{(2\pi)^4} ln[(p^2 + \overline{\eta}\eta)^2 - \lambda^2\overline{f}f] + H(\overline{a}, a) \tag{2.13}$$

where $P(\overline{f}, a, \overline{a})$ and $H(\overline{a}, a)$ are integration constants. We can impose supersymmetric boundary condition

$$V_0|_{f=0} = 0 \tag{2.14}$$

and

$$V_1|_{f=0} = 0. \tag{2.15}$$

Then we recover the effective potential, which is also directly calculated in ref.[3],

$$V_0 = \left[(ma + \frac{1}{2}\lambda a^2)f + h.c.\right] - \overline{f}f \tag{2.16}$$

and

$$V_1 = \frac{1}{2}\int \frac{d^4p}{(2\pi)^4} ln\left[1 - \frac{\lambda^2\overline{f}f}{(p^2 + \overline{\eta}\eta)^2}\right]. \tag{2.17}$$

The vacuum stability of this potential is well analyzed in ref.[3, 4]. Eq.(2.17) can be evaluated as

$$\begin{aligned} V_1 &= \frac{\pi^2}{(2\pi)^4}\left\{-\frac{1}{2}\lambda^2|f|^2\left(ln\Lambda^2 + \frac{1}{2}\right)\right. \\ &\quad \left. +\frac{1}{2}|f|^2 ln|\eta|^2 + \frac{1}{2}|\eta|^2[(1-x^2)ln(1-x) + (1-x)^2 ln(1+x)]\right\} \\ &\quad -(Z-1)|f|^2 \end{aligned} \tag{2.18}$$

where we set $x = \frac{|\lambda f|}{|\eta|^2}$, and Λ stands for the ultra-violet cut-off.

We also added the wave function renormalization factor Z(in the last term in (2.18)) in order to absorb the infinity contained in $log\Lambda^2$. In order to avoid the infrared singularity, which can appear because we set $m = 0$ in the next section, we renormalize the wave function at

$$|f| = 0 \qquad and \qquad |\eta| = M \qquad (2.19)$$

where M has the dimensions of a mass. The wave-function renormalization factor is then fixed as

$$Z = 1 - \alpha \left(ln \frac{\Lambda}{M^2} - 1 \right). \qquad (2.20)$$

The total effective potential up to one-loop level is now given by

$$
\begin{aligned}
V_{eff} &= -|f|^2 \left(1 - \alpha ln \frac{|\eta|^2}{M^2\lambda^2} \right) \\
&\quad + \frac{\alpha |\eta|^4}{2\lambda^2}[(1+x)^2 ln(1+x) + (1-x)^2 ln(1-x) - 3x^2] \\
&\quad + \lambda[(a_1^2 - a_2^2)f_1 + 2a_1 a_2 f_2] + 2m(a_1 f_1 + a_2 f_2). \qquad (2.21)
\end{aligned}
$$

Here we set

$$
\begin{cases}
f = f_1 + if_2 \\
a = a_1 - ia_2
\end{cases},
$$

$$\alpha = \frac{\pi^2\lambda^2}{2(2\pi)^4} \ , \quad x = \frac{|\lambda f|}{|\eta|^2}.$$

In order to discuss the vacuum stability, we parametrize f_1 and f_2 by

$$tan\beta = \frac{f_1}{f_2} \qquad (2.22)$$

and evaluate V_{eff} at $\frac{\partial V_{eff}}{\partial \beta} = 0$ (This corresponds to the direction of the valley of the effective potential). We then find

$$
\begin{aligned}
V_{eff} &= -\frac{|\eta|^4 x^2}{\lambda^2} \left(1 - \alpha ln \frac{|\eta|^2}{M^2} \right) + \frac{\alpha}{2}|\eta|^4[(1+x)^2 ln(1+x) + (1-x)^2 ln(1-x) - 3x^2] \\
&\quad + \frac{x|a||\eta|^2}{\lambda}\sqrt{\lambda^2(a_1^2 + a_2^2) + 2m\lambda a_1 + m^2} \qquad (2.23)
\end{aligned}
$$

To take account of the two possible signs of the square-root, we extend the range of x to $-\infty < x < +\infty$. This potential develops an imaginary part for $|x| > 1$ and this means that the solution

$$|f| \neq 0 \qquad and \qquad |\eta| = 0 \qquad (2.24)$$

is dynamically unstable. We can find the stationary value of this effective potential in the region $|x| \leq 1$ assuming that α is small. The effective potential can be written as

$$V_{eff} \cong -\frac{|\eta|^4 x^2}{\lambda^2}\left(1 - \alpha ln\frac{|\eta|^2}{M^2}\right)$$
$$+\frac{x|a||\eta|^2}{\lambda}\sqrt{\lambda^2(a_1^2 + a_2^2) + 2m\lambda a_1 + m^2}. \tag{2.25}$$

Taking the minimum of the potential $(\partial V_{eff}/\partial x = 0)$, we obtain

$$V_{eff} = \frac{|a|^2}{4}\frac{\lambda^2(a_1^2 + a_2^2) + 2m\lambda a_1 + m^2}{1 - \alpha ln\frac{|\eta|^2}{M^2}} \tag{2.26}$$

for

$$x = \frac{1}{2}\frac{\lambda|a|\sqrt{\lambda^2(a_1^2 + a_2^2) + 2m\lambda a_1 + m^2}}{|\eta|^2\left(1 - \alpha ln\frac{|\eta|^2}{M^2}\right)} \tag{2.27}$$

This potential has its minimum at

$$a_1 = 0, \qquad a_2 = 0, \quad and \quad f = 0$$

or

$$a_1 = -\frac{m}{\lambda}, \qquad a_2 = 0, \quad and \quad f = 0. \tag{2.28}$$

In both solutions, f is zero and supersymmetry is not broken. The second solution gives non-zero vev of a but f still remains zero: Two solutions (2.28) are actually two stable physically equivalent solutions, since one can pass from one to the other by a redefinition of the fields[2]. When we consider the massless Wess-Zumino model in the next section, the second solution becomes $a_1 = 0, a_2 = 0$ so the vev of a remains zero. Detailed study of this phenomenon from another point of view is given in ref.[2].

Let us examine the physical meanings of this solution. At the tree level, the equation of motion for auxiliary field is

$$F = \frac{1}{2}\lambda\overline{A}^2. \tag{2.29}$$

At the first glance, this equation seems to suggest that if the tree level potential develops a non-zero vacuum expectation value$< \overline{A} >$, $< F >$ becomes non-zero and the supersymmetry of the theory can be broken spontaneously. But this does not happen. Including higher order quantum corrections, supersymmetry-breaking vacuum ($< F >= \frac{1}{2}\lambda < \overline{A} >^2$

and$< \overline{A} >$ is non-zero) becomes unstable and the supersymmetric vacuum ($< F >= 0$) remains stable.

Furthermore, there is no Λ dependence in the effective potential after renormalization of the wave-function.

To analyze the behavior of the effective potential at small $|\eta|$ reliably, the renormalization group improvement of the effective potential has also been discussed in ref.[3]. The effective potential for the massless theory is

$$
\begin{aligned}
V_{eff} &= -\frac{|\eta|^4 x^2}{\lambda^2}\left(1 - \alpha ln\frac{|\eta|^2}{M^2}\right) + \frac{\alpha}{2}|\eta|^4[(1+x)^2 ln(1+x) + (1-x)^2 ln(1-x) - 3x^2] \\
&\quad + x|a|^2|\eta|^2
\end{aligned}
\tag{2.30}
$$

The stationary value of this potential in this region $|x| < 1$ is estimated to be

$$
V_{eff} = \frac{\lambda^2|a|^4}{4\left(1 - \alpha ln\frac{|\eta|^2}{M^2}\right)}
\tag{2.31}
$$

at

$$
x = \frac{\lambda|a|^2}{|\eta|^2\left(1 - \alpha ln\frac{|\eta|^2}{M^2}\right)}.
\tag{2.32}
$$

Renormalization group improvement of V_{eff} suggests that

$$
\begin{aligned}
V_{eff} &\simeq \frac{1}{4}(\lambda(M)|a|^3)^{\frac{4}{3}}\lambda(|a|)^{\frac{2}{3}} \\
&\simeq \frac{1}{4}(\lambda(M))^2|a|^4 \frac{1}{\left(1 - 3\alpha ln\frac{|\eta|^2}{M^2}\right)^{\frac{1}{3}}}
\end{aligned}
\tag{2.33}
$$

with the running coupling

$$
\lambda(|a|) = \frac{\lambda(M)}{\left[1 - 3\alpha ln\frac{|a|^2}{M^2}\right]^{\frac{1}{2}}}.
\tag{2.34}
$$

Note that the combination $\lambda(M)|a|^3$ is renormalization group invariant in this theory.

V_{eff} in (2.33) has a minimum at $|a| = 0$ for which $\lambda(|a|) \to 0$ and the analysis of V_{eff} is reliable. For $|a| \to 0$, $x \to 0$ in (2.33) and thus $|f| \to 0$ and no supersymmetry breaking.

This explicit analysis, which is useful to the discussion in the next section, is of course consistent with the analysis on the basis of Witten index[6].

For the discussion of the next section, we summarize the results restricting ourselves to the massless Wess-Zumino model. First, there is no supersymmetry-breaking vacuum. Second, the vev of scalar field A remains zero.

3 The meaning of NJL method in WZ model

In this section we re-examine the physical backgrounds of the NJL method proposed in ref.[1]. For convenience, we first recapitulate the basic procedure in ref.[1].

The same lagrangian (2.1) is used, but at the first stage we eliminate the auxiliary field F using the equation of motion. The result is (we here set $m = 0$)

$$L = i\partial_m \overline{\psi} \overline{\sigma}^m \psi + A^* \Box A - [\frac{\lambda}{2} \psi\psi A + h.c.] - \frac{1}{4}\lambda^2 |A|^4. \tag{3.1}$$

The equations of motion are given by

$$
\begin{cases}
\Box A + \frac{1}{2}\lambda^2 A^* AA + \frac{1}{2}\lambda\overline{\psi\psi} &=\ 0 \\
\Box A^* + \frac{1}{2}\lambda^2 A^* A^* A + \frac{1}{2}\lambda\psi\psi &=\ 0 \\
[i\partial_m \overline{\sigma}^m - \lambda A]\,\psi &=\ 0
\end{cases} \tag{3.2}
$$

Taking the vacuum expectation value of the first equation in (3.2), one obtains

$$\Box <A> +\frac{1}{2}\lambda^2 <AAA^*> = -\frac{1}{2}\lambda <\overline{\psi\psi}> . \tag{3.3}$$

Expansion of $<A^*AA>$ and $<\overline{\psi\psi}>$ to the one-loop level(i.e., to the order of $\hbar$) is given by

$$
\begin{cases}
<A^*AA> &=\ <A^*><A><A> + <A^*>\left[\text{-}AAloop\right] + <A>\left[\text{-}AA^*loop\right] \\
<\overline{\psi\psi}> &=\ \left[\text{-}\overline{\psi\psi}loop\right].
\end{cases} \tag{3.4}
$$

Here the results of the one-loop diagrams are symbolically represented. Then eq.(3.3) becomes, to the one-loop order,

$$
\begin{aligned}
0 =\ & \Box a + \frac{\lambda^2}{2}aaa^* \\
& +\frac{\lambda^2}{2}a\left[\text{-}AA^*loop\right] + \frac{\lambda^2}{2}a^*\left[\text{-}AAloop\right] + \frac{\lambda}{2}\left[\text{-}\overline{\psi\psi}loop\right]
\end{aligned} \tag{3.5}
$$

Neglecting the tadpoles of the bosonic fields and setting $\Box a = 0$ in (3.5), we get the same answer as in ref.[1];

$$\lambda aaa^* + Tr\left[\frac{\lambda}{2}\int \frac{d^4p}{(2\pi)^4} \frac{1}{i\partial_m \overline{\sigma}^m - \lambda a^*}\right] = 0. \tag{3.6}$$

which leads to the fermion pair condensation and a mass gap between the supersymmetric partners[1]. In fact, the above equation (3.6) can be rewritten as

$$|a|^2 = 4 \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - \lambda^2 |a|^2}. \tag{3.7}$$

This equation looks like a well-known mass-gap equation. The integration requires an ultra-violet cut-off, so the solution (a) of the self-consistent equation(3.7) depends on the ultra-violet cut-off parameter. Shifting the fields in the lagrangian as $A \to A + a$, with a given by eq.(3.7), we obtain the masses

$$
\begin{aligned}
m_A^2 &= \frac{\lambda^2}{2}|a|^2 \\
m_\psi &= \lambda|a|.
\end{aligned}
\tag{3.8}
$$

The supersymmetric partners thus appears to acquire different masses. This is the mechanism noted in ref.[1].

But we must not neglect bosonic tadpoles. As discussed in the previous section, the neglect of bosonic tadpoles in (3.5) is not consistent with the expansion in $\hbar$ and the resulting effective potential corresponds to the expansion around an unstable vacuum (i.e., $x = \frac{1}{2}$ in (2.23)). The meaning of the equation (3.3) is now clear: This equation means that the derivative of the effective potential is set to zero at the minimum, i.e., $\frac{\partial(V^0+V^{one-loop})}{\partial a^*}|_{vac} = 0$. One can easily obtain (3.3) by applying the tadpole method (2.9) to the variable a, not to f. Substituting A in (3.1) as $A \to A + a$ and using the tadpole method, one obtains

$$
\begin{aligned}
\frac{d(V_0 + V_1)}{da^*} &= \frac{\lambda^2}{2}aaa^* \\
&+ \frac{1}{2}\lambda \left[-\overline{\psi\psi}loop \right] + \frac{1}{2}\lambda^2 a \left[-AA^*loop \right] + \frac{1}{2}\lambda^2 a^* \left[-AAloop \right]
\end{aligned}
\tag{3.9}
$$

The evaluation and integration of (3.9) is slightly complicated in the present calculational scheme but the result is the same as (2.30)(see ref.[4]). Of course, there is no cutoff dependence in the final result which explicitly remains in the analysis of ref.[1], nor supersymmetry breaking induced by fermion pair condensation in the full effective potential resulting from (3.9). The stationary point of the effective potential correspond to the supersymmetry preserving point of (2.30).

In conclusion, we have shown that the supersymmetry breaking solution in ref.[1] is a direct consequence of the neglect of one-loop bosonic effects in the loop expansion of the effective potential. Since no dynamical mechanism why the one-loop fermion effects should be retained and why the one-loop boson effects should be neglected is given in ref.[1], we conclude that the so-called Nambu-Jona-Lasinio mechanism suggested there is not justified in the conventional framework of field theory without assuming some special attractive force between fermions.

Acknowledgment

References

[1] He-Shan Song and Guang-Nan Xu, J. Phys. A26 (1993) 2699
    He-Shan Song, Guang-Nan Xu and Ying An Chin, J. Phys. A26 (1993) 4463
    He-Shan Song and Guang-Nan Xu, J. Phys. A25 (1992) 4941

[2] J.Iliopoulos and B.Zumino, Nucl.Phys.B76(1974)310

[3] K.Fujikawa and W.Lang, Nucl.Phys.B88(1975)77

[4] R.Miller, Phys.Lett.124B(1983)59, Nucl.Phys.B241(1984)535

[5] S.Weinberg, Phys.Rev.D7(1973)2887

[6] E.Witten, Nucl.Phys. B202(1982)253